To find the general solution of $P(D)y=0$ where $P(D)$ is a polynomial differential operator. We find its auxiliary polynomial $P(r)$ first, Suppose

$$P(r) = (r-r_1)^{m_1}(r-r_2)^{m_2}\cdots(r-r_k)^{m_k}$$

where $r_1, r_2 \ldots r_k$ are distinct roots of $P(r)=0$

m_i is the multiplicity of root r_i & $m_1+m_2+\cdots+m_k = n$

① If r_i is real, then the m_i functions $\boxed{e^{r_i x}, xe^{r_i x}, \ldots, x^{m_i-1}e^{r_i x}}$ are L.I. solutions to $P(D)y=0$ corresponding to root r_i.

② If r_j is complex, say $r_j = a+bi$ (a, b are real & $b \neq 0$) its conjugate $a-bi$ is also a complex root of $P(r)=0$, counting multiplicities of $a+bi$ and $a-bi$, the number is $2m_j$. then the $2m_j$ functions

$$\boxed{\begin{array}{l} e^{ax}\cos bx, \ xe^{ax}\cos bx, \ \ldots, \ x^{m_j-1}e^{ax}\cos bx, \\ e^{ax}\sin bx, \ xe^{ax}\sin bx, \ \ldots, \ x^{m_j-1}e^{ax}\sin bx \end{array}}$$

are L.I. functions to $P(D)y=0$ corresponding to roots $a \pm bi$

③ the general solution to $P(D)y=0$ is the linear combination of all the n L.I. functions from all the roots of $P(r)=0$.

$$y(x) = C_1 y_1(x) + C_2 y_2(x) + \cdots + C_n y_n(x)$$